

PARI-GP Reference Card

(PARI-GP version 2.6.1)

Note: optional arguments are surrounded by braces {}.

To start the calculator, type its name in the terminal: **gp**

To exit **gp**, type **quit**, **\q**, or **<C-D>** at prompt.

Help

describe function	? <i>function</i>
extended description	?? <i>keyword</i>
list of relevant help topics	??? <i>pattern</i>

Input/Output

previous result, the result before	%, %', %'', etc.
<i>n</i> -th result since startup	% <i>n</i>
separate multiple statements on line	;
extend statement on additional lines	\
extend statements on several lines	{ <i>seq</i> ₁ ; <i>seq</i> ₂ ;
comment	/* ... */
one-line comment, rest of line ignored	\ \ ...

Metacommands & Defaults

set default <i>d</i> to <i>val</i>	default({ <i>d</i> }, { <i>val</i> }, { <i>flag</i> })
toggle timer on/off	#
print time for last result	##
print defaults	\d
set debug level to <i>n</i>	\g <i>n</i>
set memory debug level to <i>n</i>	\gm <i>n</i>
set output mode (raw=0, default=1)	\o <i>n</i>
set <i>n</i> significant digits	\p <i>n</i>
set <i>n</i> terms in series	\ps <i>n</i>
quit GP	\q
print the list of PARI types	\t
print the list of user-defined functions	\u
read file into GP	\r <i>filename</i>

Debugger / break loop

get out of break loop	break or <C-D>
go up <i>n</i> frames	dbg_up({ <i>n</i> })
examine object <i>o</i>	dbg_x(<i>o</i>)

PARI Types & Input Formats

t_INT/t_REAL. Integers, Reals	$\pm n$, $\pm n.ddd$
t_INTMOD. Integers modulo <i>m</i>	Mod(<i>n</i> , <i>m</i>)
t_FRAC. Rational Numbers	<i>n</i> / <i>m</i>
t_FFELT. Elt in finite field F_q	ffgen(q)
t_COMPLEX. Complex Numbers	$x + y * I$
t_PADIC. <i>p</i> -adic Numbers	$x + O(p^k)$
t_QUAD. Quadratic Numbers	$x + y * \text{quadgen}(D)$
t_POLMOD. Polynomials modulo <i>g</i>	Mod(<i>f</i> , <i>g</i>)
t_POL. Polynomials	$a * x^n + \dots + b$
t_SER. Power Series	$f + O(x^k)$
t_QFI/t_QFR. Imag/Real bin. quad. forms	Qfb(<i>a</i> , <i>b</i> , <i>c</i> , { <i>d</i> })
t_RFRAC. Rational Functions	<i>f</i> / <i>g</i>
t_VEC/t_COL. Row/Column Vectors	[<i>x</i> , <i>y</i> , <i>z</i>], [<i>x</i> , <i>y</i> , <i>z</i>]~
t_MAT. Matrices	[<i>x</i> , <i>y</i> ; <i>z</i> , <i>t</i> ; <i>u</i> , <i>v</i>]
t_LIST. Lists	List([<i>x</i> , <i>y</i> , <i>z</i>])
t_STR. Strings	"abc"

Reserved Variable Names

$\pi = 3.14\dots$, $\gamma = 0.57\dots$, $C = 0.91\dots$	Pi, Euler, Catalan
square root of -1	I
big-oh notation	O

Information about an Object

PARI type of object <i>x</i>	type(<i>x</i>)
length of <i>x</i> / size of <i>x</i> in memory	# <i>x</i> , sizebyte(<i>x</i>)
real or <i>p</i> -adic precision of <i>x</i>	precision(<i>x</i>), padicprec

Operators

basic operations	+, −, *, /, ^
i=i+1, i=i-1, i=i*j, ...	i++, i--, i*=j, ...
euclidean quotient, remainder	$x \backslash y$, $x \setminus y$, % <i>y</i> , divrem(<i>x</i> , <i>y</i>)
shift <i>x</i> left or right <i>n</i> bits	$x < < n$, $x > > n$ or shift(<i>x</i> , $\pm n$)
comparison operators	<=, <, >=, >, ==, !=, ==, lex, cmp
boolean operators (or, and, not)	, &&, !
bit operations	bitand, bitneg, bitor, bitxor
sign of $x = -1, 0, 1$	sign(<i>x</i>)
maximum/minimum of <i>x</i> and <i>y</i>	max, min(<i>x</i> , <i>y</i>)
integer or real factorial of <i>x</i>	<i>x</i> ! or factorial(<i>x</i>)
derivative of <i>f</i> w.r.t. <i>x</i>	<i>f</i> '
apply differential operator	diffop
restore <i>x</i> as a formal variable	<i>x</i> =' <i>x</i>
simultaneous assignment $x \leftarrow v_1, y \leftarrow v_2$	[<i>x</i> , <i>y</i>] = v

Select Components

<i>n</i> -th component of <i>x</i>	component(<i>x</i> , <i>n</i>)
<i>n</i> -th component of vector/list <i>x</i>	<i>x</i> [<i>n</i>]
components <i>a</i> , <i>a</i> + 1, ..., <i>b</i> of vector <i>x</i>	<i>x</i> [<i>a</i> .. <i>b</i>]
(<i>m</i> , <i>n</i>)-th component of matrix <i>x</i>	<i>x</i> [<i>m</i> , <i>n</i>]
row <i>m</i> or column <i>n</i> of matrix <i>x</i>	<i>x</i> [<i>m</i> ,], <i>x</i> [, <i>n</i>]
numerator/denominator of <i>x</i>	numerator(<i>x</i>), denominator

Conversions

to vector, matrix, set, list, string	Col/Vec, Mat, Set, List, Str
create PARI object (<i>x</i> mod <i>y</i>)	Mod(<i>x</i> , <i>y</i>)
make <i>x</i> a polynomial of <i>v</i>	Pol(<i>x</i> , { <i>v</i> })
as Pol/Vec, starting with constant term	Polrev, Vecrev
make <i>x</i> a power series of <i>v</i>	Ser(<i>x</i> , { <i>v</i> })
string from bytes / from format+args	Strchr, Strprintf
convert <i>x</i> to simplest possible type	simplify(<i>x</i>)
object <i>x</i> with precision <i>n</i>	precision(<i>x</i> , <i>n</i>)

Conjugates and Lifts

conjugate of a number <i>x</i>	conj(<i>x</i>)
conjugate vector of algebraic number <i>x</i>	conjvec(<i>x</i>)
norm of <i>x</i> , product with conjugate	norm(<i>x</i>)
square of L^2 norm of vector <i>x</i>	norml2(<i>x</i>)
lift of <i>x</i> from Mods	lift, centerlift(<i>x</i>)

Lists, Sets & Sorting

sort <i>x</i> by <i>k</i> -th component	vecsort(<i>x</i> , { <i>k</i> }, { <i>fl</i> = 0})
min. <i>m</i> of <i>x</i> ($m = x[i]$), max.	vecmin(<i>x</i> , {& <i>i</i> }), vecmax
does <i>y</i> belong to <i>x</i> , sorted wrt. <i>f</i>	vecsearch(<i>x</i> , <i>y</i> , { <i>f</i> })
Sets (= row vector of strings with strictly increasing entries)	
intersection of sets <i>x</i> and <i>y</i>	setintersect(<i>x</i> , <i>y</i>)
set of elements in <i>x</i> not belonging to <i>y</i>	setminus(<i>x</i> , <i>y</i>)
union of sets <i>x</i> and <i>y</i>	setunion(<i>x</i> , <i>y</i>)
does <i>y</i> belong to the set <i>x</i>	setsearch(<i>x</i> , <i>y</i> , { <i>flag</i> })
is <i>x</i> a set ?	setisset(<i>x</i>)
Lists. create empty list: $L = \text{List}()$	
append <i>x</i> to list <i>L</i>	listput(<i>L</i> , <i>x</i> , { <i>i</i> })
remove <i>i</i> -th component from list <i>L</i>	listpop(<i>L</i> , { <i>i</i> })
insert <i>x</i> in list <i>L</i> at position <i>i</i>	listinsert(<i>L</i> , <i>x</i> , <i>i</i>)
sort the list <i>L</i> in place	listsort(<i>L</i> , { <i>flag</i> })

Programming

Functions and closures

fun(vars) = my(local vars); *seq*

fun = (vars) -> my(local vars); *seq*

Control Statements (*X*: formal parameter in expression *seq*)

eval. <i>seq</i> for $a \leq X \leq b$	for(<i>X</i> = <i>a</i> , <i>b</i> , <i>seq</i>)
eval. <i>seq</i> for <i>X</i> dividing <i>n</i>	fordiv(<i>n</i> , <i>X</i> , <i>seq</i>)
eval. <i>seq</i> for primes $a \leq X \leq b$	forprime(<i>X</i> = <i>a</i> , <i>b</i> , <i>seq</i>)
eval. <i>seq</i> for $a \leq X \leq b$ stepping <i>s</i>	forstep(<i>X</i> = <i>a</i> , <i>b</i> , <i>s</i> , <i>seq</i>)
multivariable for	forvec(<i>X</i> = <i>v</i> , <i>seq</i>)
loop over partitions of <i>n</i>	forpart(<i>p</i> = <i>n</i> <i>seq</i>)
loop over vectors $v, q(v) \leq B, q > 0$	forqfvec(<i>v</i> , <i>q</i> , <i>b</i> , <i>seq</i>)
loop over subgrps <i>H</i> of abelian grp <i>G</i>	forsubgroup(<i>H</i> = <i>G</i>)
evaluate <i>seq</i> until $a \neq 0$	until(<i>a</i> , <i>seq</i>)
while $a \neq 0$, evaluate <i>seq</i>	while(<i>a</i> , <i>seq</i>)
exit <i>n</i> innermost enclosing loops	break({ <i>n</i> })
start new iteration of <i>n</i> -th enclosing loop	next({ <i>n</i> })
return <i>x</i> from current subroutine	return({ <i>x</i> })
raise an exception	error()
if $a \neq 0$, evaluate <i>seq</i> ₁ , else <i>seq</i> ₂	if(<i>a</i> , { <i>seq</i> ₁ }, { <i>seq</i> ₂ })
try <i>seq</i> ₁ , evaluate <i>seq</i> ₂ on error	iferr(<i>seq</i> ₁ , <i>E</i> , <i>seq</i> ₂)
select from <i>v</i> according to <i>f</i>	select(<i>f</i> , <i>v</i>)
apply <i>f</i> to all entries in <i>v</i>	apply(<i>f</i> , <i>v</i>)

Input/Output

print with/without \n, T _E X format	print, print1, printtex
formatted printing	printf()
write <i>args</i> to file	write, writel, writetex(<i>file</i> , <i>args</i>)
write <i>x</i> in binary format	writebin(<i>file</i> , <i>x</i>)
read file into GP	read({ <i>file</i> })
read file, return as vector of lines	readvec({ <i>file</i> })
read a string from keyboard	input()

Interface with User and System

allocates a new stack of <i>s</i> bytes	allocatemem({ <i>s</i> })
alias <i>old</i> to <i>new</i>	alias(<i>new</i> , <i>old</i>)
install function from library	install(<i>f</i> , <i>code</i> , { <i>gpf</i> }, { <i>lib</i> })
execute system command <i>a</i>	system(<i>a</i>)
as above, feed result to GP	extern(<i>a</i>)
as above, return GP string	externstr(<i>a</i>)
get \$VAR from environment	getenv("VAR")
measure time in ms.	gettime()
timeout command after <i>s</i> seconds	alarm(<i>s</i> , <i>expr</i>)

Iterations, Sums & Products

numerical integration	intnum(<i>X</i> = <i>a</i> , <i>b</i> , <i>expr</i> , { <i>flag</i> })
sum <i>expr</i> over divisors of <i>n</i>	sumdiv(<i>n</i> , <i>X</i> , <i>expr</i>)
sumdiv, with <i>expr</i> multiplicative	sumdivmult(<i>n</i> , <i>X</i> , <i>expr</i>)
sum $X = a$ to $X = b$, initialized at <i>x</i>	sum(<i>X</i> = <i>a</i> , <i>b</i> , <i>expr</i> , { <i>x</i> })
sum of series <i>expr</i>	suminf(<i>X</i> = <i>a</i> , <i>expr</i>)
sum of alternating/positive series	sumalt, sumpos
sum of series using intnum	sumnum
product $a \leq X \leq b$, initialized at <i>x</i>	prod(<i>X</i> = <i>a</i> , <i>b</i> , <i>expr</i> , { <i>x</i> })
product over primes $a \leq X \leq b$	prodeuler(<i>X</i> = <i>a</i> , <i>b</i> , <i>expr</i>)
infinite product $a \leq X \leq \infty$	prodinf(<i>X</i> = <i>a</i> , <i>expr</i>)
real root of <i>expr</i> between <i>a</i> and <i>b</i>	solve(<i>X</i> = <i>a</i> , <i>b</i> , <i>expr</i>)

Random Numbers

random integer/prime in $[0, N[$	random(<i>N</i>), randomprime
get/set random seed	getrand, setrand(<i>s</i>)

Vectors & Matrices

dimensions of matrix x	<code>matsize(x)</code>
concatenation of x and y	<code>concat($x, \{y\}$)</code>
extract components of x	<code>vecextract($x, y, \{z\}$)</code>
transpose of vector or matrix x	<code>mattranspose(x)</code> or <code>x-</code> <code>matadjoint(x)</code>
eigenvectors/values of matrix x	<code>mateigen(x)</code>
characteristic/minimal polynomial of x	<code>charpoly(x)</code> , <code>minpoly</code>
trace/determinant of matrix x	<code>trace(x)</code> , <code>matdet</code>
Frobenius form of x	<code>matfrobenius(x)</code>
QR decomposition	<code>matqr(x)</code>

Constructors & Special Matrices

row vec. of $expr$ eval'd at $1 \leq i \leq n$	<code>vector($n, \{i\}, \{expr\}$)</code>
col. vec. of $expr$ eval'd at $1 \leq i \leq n$	<code>vectorv($n, \{i\}, \{expr\}$)</code>
matrix $1 \leq i \leq m, 1 \leq j \leq n$	<code>matrix($m, n, \{i\}, \{j\}, \{expr\}$)</code>
define matrix by blocks	<code>matconcat(B)</code>
diagonal matrix with diagonal x	<code>matdiagonal(x)</code>
$n \times n$ identity matrix	<code>matid(n)</code>
Hessenberg form of square matrix x	<code>mathess(x)</code>
$n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$	<code>mathilbert(n)</code>
companion matrix to polynomial x	<code>matcompanion(x)</code>
Sylvester matrix of x	<code>polsylvestermatrix(x)</code>

Gaussian elimination

kernel of matrix x	<code>matker($x, \{flag\}$)</code>
intersection of column spaces of x and y	<code>matintersect(x, y)</code>
solve $M * X = B$ (M invertible)	<code>matsolve(M, B)</code>
as solve, modulo D (col. vector)	<code>matolvemod(M, D, B)</code>
one sol of $M * X = B$	<code>matinverseimage(M, B)</code>
basis for image of matrix x	<code>matimage(x)</code>
supplement columns of x to get basis	<code>mat supplement(x)</code>
rows, cols to extract invertible matrix	<code>matindexrank(x)</code>
rank of the matrix x	<code>matrank(x)</code>

Lattices & Quadratic Forms

upper triangular Hermite Normal Form	<code>mathnf(x)</code>
HNF of x where d is a multiple of $\det(x)$	<code>mathnfmod(x, d)</code>
elementary divisors of x	<code>matsnf(x)</code>
LLL-algorithm applied to columns of x	<code>qflll($x, \{flag\}$)</code>
like <code>qflll</code> , x is Gram matrix of lattice	<code>qflllgram($x, \{flag\}$)</code>
LLL-reduced basis for kernel of x	<code>matkerint(x)</code>
\mathbf{Z} -lattice \longleftrightarrow \mathbf{Q} -vector space	<code>matrixqz(x, p)</code>
signature of quad form ${}^t y * x * y$	<code>qf sign(x)</code>
decomp into squares of ${}^t y * x * y$	<code>qfgaussred(x)</code>
eigenvals/eigenvecs for real symmetric x	<code>qfjacobi(x)</code>
find up to m sols of ${}^t y * x * y \leq b$	<code>qfminim(x, b, m)</code>
perfection rank of x	<code>qfperfection(x)</code>
$v, v[i] :=$ number of sols of ${}^t y * x * y = i$	<code>qfrep($x, B, \{flag\}$)</code>
automorphism group of q	<code>qfauto(q)</code>
find isomorphism between q and Q	<code>qfisom(q, Q)</code>

Formal & p-adic Series

truncate power series or p -adic number	<code>truncate(x)</code>
valuation of x at p	<code>valuation(x, p)</code>

Dirichlet and Power Series

Taylor expansion around 0 of f w.r.t. x	<code>taylor(f, x)</code>
$\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$	<code>serconvol(a, b)</code>
$f = \sum a_k t^k$ from $\sum (a_k / k!) t^k$	<code>serlaplace(f)</code>
reverse power series F so $F(f(x)) = x$	<code>serreverse(f)</code>
Dirichlet series multiplication / division	<code>dirmul, dirdiv(x, y)</code>
Dirichlet Euler product (b terms)	<code>direuler($p = a, b, expr$)</code>

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Polynomials & Rational Functions

degree of f	<code>poldegree(f)</code>
coeff. of degree n of f , leading coeff.	<code>polcoeff(f, n)</code> , <code>pollead</code>
gcd of coefficients of f	<code>content(f)</code>
replace x by y	<code>subst(f, x, y)</code>
evaluate f replacing vars by their value	<code>eval(f)</code>
replace polynomial expr. $T(x)$ by y in f	<code>substpol(f, T, y)</code>
replace x_1, \dots, x_n by y_1, \dots, y_n in f	<code>substvec(f, x, y)</code>
discriminant of polynomial f	<code>poldisc(f)</code>
resultant $R = \text{Res}_v(f, g)$	<code>polresultant($f, g, \{v\}$)</code>
$[u, v, R], xu + yv = \text{Res}_v(f, g)$	<code>polresultanttext($x, y, \{v\}$)</code>
derivative of f w.r.t. x	<code>deriv($f, \{x\}$)</code>
formal integral of f w.r.t. x	<code>intformal($f, \{x\}$)</code>
formal sum of f w.r.t. x	<code>sumformal($f, \{x\}$)</code>
reciprocal poly $x^{\deg f} f(1/x)$	<code>polrecip(f)</code>
interpol. pol. eval. at a	<code>polinterpolate($X, \{Y\}, \{a\}, \{&e\}$)</code>
initialize t for Thue equation solver	<code>thueinit(f)</code>
solve Thue equation $f(x, y) = a$	<code>thue($t, a, \{sol\}$)</code>

Roots and Factorization

number of real roots of $f, a < x \leq b$	<code>polsturm($f, \{a\}, \{b\}$)</code>
complex roots of f	<code>polroots(f)</code>
symmetric powers of roots of f up to n	<code>polsym(f, n)</code>
factor f	<code>factor($f, \{lim\}$)</code>
factor $f \bmod p$ / roots	<code>factormod(f, p)</code> , <code>polrootsmod</code>
factor f over \mathbf{F}_{p^a} / roots	<code>factorff(f, p, a)</code> , <code>polrootsff</code>
factor f over \mathbf{Q}_p / roots	<code>factorpadic(f, p, r)</code> , <code>polrootspadic</code>
find irreducible $T \in \mathbf{F}_p[x], \deg T = n$	<code>ffinit($p, n, \{x\}$)</code>
$\#\{\text{monic irred. } T \in \mathbf{F}_q[x], \deg T = n\}$	<code>ffnbirred(q, n)</code>
p -adic root of f cong. to $a \bmod p$	<code>padicappr(f, a)</code>
Newton polygon of f for prime p	<code>newtonpoly(f, p)</code>
extensions of \mathbf{Q}_p of degree N	<code>padicfields(p, N)</code>

Special Polynomials

n -th cyclotomic polynomial in var. v	<code>polcyclo($n, \{v\}$)</code>
d -th degree subfield of $\mathbf{Q}(\zeta_n)$	<code>polsubcyclo($n, d, \{v\}$)</code>
$P_n, T_n/U_n, H_n$	<code>pollegendre, polchebyshev, polhermite</code>

Transcendental and p -adic Functions

real, imaginary part of x	<code>real(x)</code> , <code>imag(x)</code>
absolute value, argument of x	<code>abs(x)</code> , <code>arg(x)</code>
square/ n th root of x	<code>sqrtn(x)</code> , <code>sqrtn($x, n, \{&z\}$)</code>
trig functions	<code>sin, cos, tan, cotan</code>
inverse trig functions	<code>asin, acos, atan</code>
hyperbolic functions	<code>sinh, cosh, tanh</code>
inverse hyperbolic functions	<code>asinh, acosh, atanh</code>
exponential / natural log of x	<code>exp, log</code>
Euler Γ function, $\log \Gamma, \Gamma'/\Gamma$	<code>gamma, lngamma, psi</code>
incomplete gamma function ($y = \Gamma(s)$)	<code>incgam($s, x, \{y\}$)</code>
exponential integral $\int_x^\infty e^{-t}/t dt$	<code>eint1(x)</code>
error function $2/\sqrt{\pi} \int_x^\infty e^{-t^2} dt$	<code>erfc(x)</code>
dilogarithm of x	<code>dilog(x)</code>
m -th polylogarithm of x	<code>polylog($m, x, \{flag\}$)</code>
U -confluent hypergeometric function	<code>hyperu(a, b, u)</code>
Bessel $J_n(x), J_{n+1/2}(x)$	<code>besselj(n, x)</code> , <code>besseljh(n, x)</code>
Bessel $I_\nu, K_\nu, H_\nu^1, H_\nu^2, N_\nu$	<code>(bessel)i, k, h1, h2, n</code>
Lambert $W: x$ s.t. $xe^x = y$	<code>lambertw(y)</code>
Teichmuller character of p -adic x	<code>teichmuller(x)</code>

Elementary Arithmetic Functions

vector of binary digits of $ x $	<code>binary(x)</code>
bit number n of integer x	<code>bittest(x, n)</code>
Hamming weight of integer x	<code>hammingweight(x)</code>
ceiling/floor/fractional part	<code>ceil, floor, frac</code>
round x to nearest integer	<code>round($x, \{&e\}$)</code>
truncate x	<code>truncate($x, \{&e\}$)</code>
gcd/LCM of x and y	<code>gcd(x, y)</code> , <code>lcm(x, y)</code>
gcd of entries of a vector/matrix	<code>content(x)</code>

Primes and Factorization

add primes in v to prime table	<code>addprimes(v)</code>
Chebyshev $\pi(x)$, n -th prime p_n	<code>primepi(x)</code> , <code>prime(n)</code>
vector of first n primes	<code>primes(n)</code>
smallest prime $\geq x$	<code>nextprime(x)</code>
largest prime $\leq x$	<code>precprime(x)</code>
factorization of x	<code>factor($x, \{lim\}$)</code>
$n = df^2$, d squarefree/fundamental	<code>core($n, \{fl\}$)</code> , <code>coredisc</code>
recover x from its factorization	<code>factorback($f, \{e\}$)</code>

Divisors

number of prime divisors $\omega(n)$ / $\Omega(n)$	<code>omega(n)</code> , <code>bigomega</code>
divisors of n / number of divisors $\tau(n)$	<code>divisors(n)</code> , <code>numdiv</code>
sum of (k -th powers of) divisors of n	<code>sigma($n, \{k\}$)</code>

Special Functions and Numbers

binomial coefficient $\binom{x}{y}$	<code>binomial(x, y)</code>
Bernoulli number B_n as real/rational	<code>bernreal(n)</code> , <code>bernfrac</code>
Bernoulli polynomial $B_n(x)$	<code>bernpol($n, \{x\}$)</code>
n -th Fibonacci number	<code>fibonacci(n)</code>
Stirling numbers $s(n, k)$ and $S(n, k)$	<code>stirling($n, k, \{flag\}$)</code>
number of partitions of n	<code>numbpart(n)</code>
Möbius μ -function	<code>moebius(x)</code>
Hilbert symbol of x and y (at p)	<code>hilbert($x, y, \{p\}$)</code>
Kronecker-Legendre symbol $(\frac{x}{y})$	<code>kronecker(x, y)</code>
Dedekind sum $s(h, k)$	<code>sumdedekind(h, k)</code>

Multiplicative groups $(\mathbf{Z}/N\mathbf{Z})^*$, \mathbf{F}_q^*

Euler ϕ -function	<code>eulerphi(x)</code>
multiplicative order of x (divides o)	<code>znorder($x, \{o\}$)</code> , <code>fforder</code>
primitive root mod q / $x \bmod$	<code>znprimroot(q)</code> , <code>ffprimroot(x)</code>
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	<code>znstar(n)</code>
discrete logarithm of x in base g	<code>znlog($x, g, \{o\}$)</code> , <code>fflog</code>

Miscellaneous

integer square / n -th root of x	<code>sqrtnint(x)</code> , <code>sqrtnint(x, n)</code>
solve $z \equiv x$ and $z \equiv y$	<code>chinese(x, y)</code>
minimal u, v so $xu + yv = \gcd(x, y)$	<code>gcdext(x, y)</code>
continued fraction of x	<code>contfrac($x, \{b\}, \{lmax\}$)</code>
last convergent of continued fraction x	<code>contfracpnqn(x)</code>
rational approximation to x	<code>bestappr(x, k)</code> , <code>bestapprPade</code>

True-False Tests

is x the disc. of a quadratic field?	<code>isfundamental(x)</code>
is x a prime?	<code>isprime(x)</code>
is x a strong pseudo-prime?	<code>ispseudoprime(x)</code>
is x square-free?	<code>issquarefree(x)</code>
is x a square?	<code>issquare($x, \{&n\}$)</code>
is x a perfect power?	<code>ispower($x, \{k\}, \{&n\}$)</code>
is pol irreducible?	<code>polisirreducible(pol)</code>

Based on an earlier version by Joseph H. Silverman

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PARI-GP Reference Card (2)

(PARI-GP version 2.6.1)

Elliptic Curves

Elliptic curve initially given by 5-tuple $v = [a_1, a_2, a_3, a_4, a_6]$. Initialize *ell* struct $E = \text{ellinit}(v, \{Domain\})$. Points are $[x, y]$, the origin is $[0]$. Struct members accessed as $E.member$:

- All domains: **E.a1,a2,a3,a4,a6, b2,b4,b6,b8, c4,c6, disc, j**
 - E defined over **R** or **C**
 - x -coords. of points of order 2 **E.roots**
 - periods / quasi-periods **E.omega,E.eta**
 - volume of complex lattice **E.area**
 - E defined over **\mathbf{Q}_p**
 - residual characteristic **E.p**
 - If $|j|_p > 1$: Tate's $[u^2, u, q, [a, b]]$ **E.tate**
 - E defined over **\mathbf{F}_q**
 - characteristic **E.p**
 - $\#E(\mathbf{F}_q)$ /cyclic structure/generators **E.no, E.cyc, E.gen**
 - E defined over **\mathbf{Q}**
 - generators of $E(\mathbf{Q})$ (require **elldata**) **E.gen**
 - $[a_1, a_2, a_3, a_4, a_6]$ from j -invariant **ellfromj(j)**
 - change curve E using $v = [u, r, s, t]$ **ellchangecurve(E, v)**
 - change point z using $v = [u, r, s, t]$ **ellchangepoint(z, v)**
 - add points $P + Q / P - Q$ **elladd(E, P, Q), ellsub**
 - negate point **ellneg(E, P)**
 - compute $n \cdot z$ **ellmul(E, z, n)**
 - n -division polynomial $f_n(x)$ **elldivpol(E, n, {x})**
 - check if z is on E **ellisoncurve(E, z)**
 - order of torsion point z **ellorder(E, z)**
 - y -coordinates of point(s) for x **ellordinate(E, x)**
 - point $[\wp(z), \wp'(z)]$ corresp. to z **ellztopoint(E, z)**
 - complex z such that $p = [\wp(z), \wp'(z)]$ **ellpointtoz(E, p)**
- Curves over finite fields, Pairings**
- random point on E **random(E)**
 - $\#E(\mathbf{F}_q)$ **ellcard(E)**
 - structure $\mathbf{Z}/d_1\mathbf{Z} \times \mathbf{Z}/d_2\mathbf{Z}$ of $E(\mathbf{F}_q)$ **ellgroup(E)**
 - Weil pairing of m -torsion pts x, y **ellweilpairing(E, x, y, m)**
 - Tate pairing of x, y ; x m -torsion **elltatepairing(E, x, y, m)**
 - Discrete log, find n s.t. $P = [n]Q$ **elllog(E, P, Q, {ord})**
- Curves over \mathbf{Q} and the L -function**
- canonical bilinear form taken at z_1, z_2 **ellbil(E, z1, z2)**
 - canonical height of z **ellheight(E, z, {flag})**
 - height regulator matrix for pts in x **ellheightmatrix(E, x)**
 - cond, min mod, Tamagawa num $[N, v, c]$ **ellglobalred(E)**
 - reduction of $y^2 + Qy = P$ (genus 2) **genus2red(Q, P, {p})**
 - Kodaira type of p -fiber of E **elllocalred(E, p)**
 - minimal model of E/\mathbf{Q} **ellminimalmodel(E, {&v})**
 - p -th coeff a_p of L -function, p prime **ellap(E, p)**
 - k -th coeff a_k of L -function **ellak(E, k)**
 - vector of first n a_k 's in L -function **elllan(E, n)**
 - $L(E, s)$ **elllseries(E, s)**
 - $L^{(r)}(E, 1)$ **ellL1(E, r)**
 - return a Heegner point on E of rank 1 **ellheegner(E)**
 - order of vanishing at 1 **ellanalyticrank(E, {eps})**
 - root number for $L(E, \cdot)$ at p **ellrootno(E, {p})**
 - torsion subgroup with generators **elltors(E)**
 - modular parametrization of E **elltaniyama(E)**

Elldata package, Cremona's database:

db code \leftrightarrow *[conductor, class, index]* **ellconvertname(s)**
generators of Mordell-Weil group **ellgenerators(E)**
look up E in database **ellidentify(E)**
all curves matching criterion **ellsearch(N)**
loop over curves with cond. from a to b **forell(E, a, b, seq)**

Elliptic & Modular Functions

$w = [\omega_1, \omega_2]$ or *ell* struct **(E.omega)**, $\tau = \omega_1/\omega_2$.
arithmetic-geometric mean **agm(x, y)**
elliptic j -function $1/q + 744 + \dots$ **ellj(x)**
Weierstrass $\sigma/\wp/\zeta$ function **ellsigma(w, z), ellwp, ellzeta**
periods/quasi-periods **ellperiods(E, {flag}), elleta(w)**
 $(2i\pi/\omega_2)^k E_k(\tau)$ **elleisnum(w, k, {flag})**
modified Dedekind η func. $\prod(1 - q^n)$ **eta(x, {flag})**
Jacobi sine theta function **theta(q, z)**
 k -th derivative at $z=0$ of **theta(q, z)** **thetanullk(q, k)**
Weber's f functions **weber(x, {flag})**
Riemann's zeta $\zeta(s) = \sum n^{-s}$ **zeta(s)**

Binary Quadratic Forms

create $ax^2 + bxy + cy^2$ (distance d) **Qfb(a, b, c, {d})**
reduce x ($s = \sqrt{D}$, $t = [s]$) **qfbred(x, {flag}, {D}, {l}, {s})**
composition of forms $x*y$ or **qfbnucomp(x, y, l)**
 n -th power of form x^n or **qfbnupow(x, n)**
composition without reduction **qfbcomprow(x, y)**
 n -th power without reduction **qfbpowrow(x, n)**
prime form of disc. x above prime p **qfbprimeform(x, p)**
class number of disc. x **qfbclassno(x)**
Hurwitz class number of disc. x **qfbhclassno(x)**
Solve $Q(x, y) = p$ in integers, p prime **qfbsolve(Q, p)**

Quadratic Fields

quadratic number $\omega = \sqrt{x}$ or $(1 + \sqrt{x})/2$ **quadgen(x)**
minimal polynomial of ω **quadpoly(x)**
discriminant of **$\mathbf{Q}(\sqrt{D})$** **quaddisc(x)**
regulator of real quadratic field **quadregulator(x)**
fundamental unit in real **$\mathbf{Q}(x)$** **quadunit(x)**
class group of **$\mathbf{Q}(\sqrt{D})$** **quadclassunit(D, {flag}, {t})**
Hilbert class field of **$\mathbf{Q}(\sqrt{D})$** **quadhilbert(D, {flag})**
ray class field modulo f of **$\mathbf{Q}(\sqrt{D})$** **quadray(D, f, {flag})**

General Number Fields: Initializations

A number field K is given by a monic irreducible $f \in \mathbf{Z}[X]$.

init number field structure *nf* **nfinit(f, {flag})**

nf members:

polynomial defining *nf*, $f(\theta) = 0$ **nf.pol**
number of real/complex places **nf.r1/r2/sign**
discriminant of *nf* **nf.disc**
 T_2 matrix **nf.t2**
vector of roots of f **nf.roots**
integral basis of \mathbf{Z}_K as powers of θ **nf.zk**
different **nf.diff**
codifferent **nf.codiff**
index **nf.index**
recompute *nf* using current precision **nfnewprec(nf)**
init relative *rnf* given by $g = 0$ over K **rnfinit(nf, g)**
init *bnf* structure **bnfinit(f, {flag})**

bnf members:

same as *nf*, plus
underlying *nf* **bnf.nf**
classgroup **bnf.clgp**
regulator **bnf.reg**
fundamental units **bnf.fu**
torsion units **bnf.tu**
compute a *bnf* from small *bnf* **bnfinit(sbnf)**
add S -class group and units, yield *bnf* **bnfsunit(nf, S)**
init class field structure *bnr* **bnrinit(bnf, m, {flag})**
bnr members: same as *bnf*, plus
underlying *bnf* **bnr.bnf**
big ideal structure **bnr.bid**
modulus **bnr.mod**
structure of $(\mathbf{Z}_K/m)^*$ **bnr.zkst**

Basic Number Field Arithmetic (nf)

Elements are **t_INT, t_FRAC, t_POL, t_POLMOD**, or **t_COL** (on integral basis *nf.zk*). Basic operations (prefix **nfelt**): (**nfelt**)**add**, **mul**, **pow**, **div**, **diveuc**, **mod**, **divrem**, **val**, **trace**, **norm**
express x on integer basis **nfalttobasis(nf, x)**
express element x as a polmod **nfbasistoalg(nf, x)**
reverse polmod $a = A(X) \bmod T(X)$ **modreverse(a)**
integral basis of field def. by $f = 0$ **nfbasis(f)**
field discriminant of field $f = 0$ **nfdisc(f)**
smallest poly defining $f = 0$ (slow) **polredabs(f, {flag})**
small poly defining $f = 0$ (fast) **polredbest(f, {flag})**
are fields $f = 0$ and $g = 0$ isomorphic? **nfisism(f, g)**
is field $f = 0$ a subfield of $g = 0$? **nfisincl(f, g)**
compositum of $f = 0$, $g = 0$ **polcompositum(f, g, {flag})**
subfields (of degree d) of *nf* **nfsubfields(nf, {d})**
roots of unity in *nf* **nfrootsof1(nf)**
roots of g belonging to *nf* **nfroots({nf}, g)**
factor g in *nf* **nfactor(nf, g)**
factor g mod prime *pr* in *nf* **nfactormod(nf, g, pr)**
conjugates of a root θ of *nf* **nfgaloisconj(nf, {flag})**
apply Galois automorphism s to x **nfgaloisapply(nf, s, x)**
quadratic Hilbert symbol (at p) **nfhilbert(nf, a, b, {p})**

Linear and algebraic relations

poly of degree $\leq k$ with root $x \in \mathbf{C}$ **algdep(x, k)**
alg. dep. with pol. coeffs for series s **seralgdep(s, x, y)**
small linear rel. on coords of vector x **lindep(x)**

Dedekind Zeta Function ζ_K , Hecke L series

ζ_K as Dirichlet series, $N(I) < b$ **dirzetak(nf, b)**
init *nfz* for field $f = 0$ **zetakinit(f)**
compute $\zeta_K(s)$ **zetak(nfz, s, {flag})**
Artin root number of K **bnrrootnumber(bnr, chi, {flag})**
 $L(1, \chi)$, for all χ trivial on H **bnrL1(bnr, {H}, {flag})**

Class Groups & Units (bnf, bnr)

$a_1, \{a_2\}, \{a_3\}$ usually *bnr, subgp* or *bnf, module, {subgp}*
remove GRH assumption from *bnf* **bnfcertify(bnf)**
expo. of ideal x on class gp **bnfisprincipal(bnf, x, {flag})**
expo. of ideal x on ray class gp **bnrisprincipal(bnr, x, {flag})**
expo. of x on fund. units **bnfisunit(bnf, x)**
as above for S -units **bnfissunit(bnf, x)**
signs of real embeddings of *bnf.fu* **bnfsignunit(bnf)**
narrow class group **bnfnarrow(bnf)**

Class Field Theory

ray class number for mod. m bnrclassno(bnf, m)
discriminant of class field ext bnrdisc($a_1, \{a_2\}, \{a_3\}$)
ray class numbers, l list of mods bnrclassnolist(bnf, l)
discriminants of class fields bnrdisclist($bnf, l, \{arch\}, \{flag\}$)
decode output from bnrdisclist bnfdecodemodule(nf, fa)
is modulus the conductor? bnrisconductor($a_1, \{a_2\}, \{a_3\}$)
conductor of character chi bnrconductorofchar(bnr, chi)
conductor of extension bnrconductor($a_1, \{a_2\}, \{a_3\}, \{flag\}$)
conductor of extension def. by g rnfconductor(bnf, g)
Artin group of ext. def'd by g rnfnormgroup(bnr, g)
subgroups of bnr , index $\leq b$ subgrouplist($bnr, b, \{flag\}$)
rel. eq. for class field def'd by sub rnfkummer($bnr, sub, \{d\}$)
same, using Stark units (real field) bnrstark($bnr, sub, \{flag\}$)

Ideals: elements, primes, or matrix of generators in HNF
is id an ideal in nf ? nfnisideal(nf, id)
is x principal in bnf ? bnfnisprincipal(bnf, x)
give $[a, b]$, s.t. $a\mathbf{Z}_K + b\mathbf{Z}_K = x$ idealtwoelt($nf, x, \{a\}$)
put ideal $a(a\mathbf{Z}_K + b\mathbf{Z}_K)$ in HNF form idealhnf($nf, a, \{b\}$)
norm of ideal x idealnrm(nf, x)
minimum of ideal x (direction v) idealmin(nf, x, v)
LLL-reduce the ideal x (direction v) idealred($nf, x, \{v\}$)

Ideal Operations

add ideals x and y idealadd(nf, x, y)
multiply ideals x and y idealmul($nf, x, y, \{flag\}$)
intersection of ideals x and y idealintersect($nf, x, y, \{flag\}$)
 n -th power of ideal x idealpow($nf, x, n, \{flag\}$)
inverse of ideal x idealinv(nf, x)
divide ideal x by y idealdiv($nf, x, y, \{flag\}$)
Find $(a, b) \in x \times y, a + b = 1$ idealaddtoone($nf, x, \{y\}$)
coprime integral A, B such that $x = A/B$ idealnumden(nf, x)

Primes and Multiplicative Structure

factor ideal x in nf idealfactor(nf, x)
expand ideal factorization in nf idealfactorback(nf, f, e)
decomposition of prime p in nf idealprimedec(nf, p)
valuation of x at prime ideal pr idealval(nf, x, pr)
weak approximation theorem in nf idealchinese(nf, x, y)
give bid =structure of $(\mathbf{Z}_K/id)^*$ idealstar($nf, id, \{flag\}$)
discrete log of x in $(\mathbf{Z}_K/bid)^*$ ideallog(nf, x, bid)
idealstar of all ideals of norm $\leq b$ ideallist($nf, b, \{flag\}$)
add Archimedean places ideallistarch($nf, b, \{ar\}, \{flag\}$)
init **prmod** structure nfmodprinit(nf, pr)
kernel of matrix M in $(\mathbf{Z}_K/pr)^*$ nfkernelpr($nf, M, prmod$)
solve $Mx = B$ in $(\mathbf{Z}_K/pr)^*$ nfsolvemodpr($nf, M, B, prmod$)

Galois theory over \mathbf{Q}

Galois group of field $\mathbf{Q}[x]/(f)$ polgalois(f)
initializes a Galois group structure G galoisinit($pol, \{den\}$)
action of p in nfgaloisconj form galoispermtopol($G, \{p\}$)
identify as abstract group galoisidentify(G)
export a group for GAP/MAGMA galoisexport($G, \{flag\}$)
subgroups of the Galois group G galoissubgroups(G)
is subgroup H normal? galoisisnormal(G, H)
subfields from subgroups galoissubfields($G, \{flag\}, \{v\}$)
fixed field galoisfixedfield($G, perm, \{flag\}, \{v\}$)
Frobenius at maximal ideal P idealfrobenius(nf, G, P)
ramification groups at P idealramgroups(nf, G, P)

PARI-GP Reference Card (2)

(PARI-GP version 2.6.1)

is G abelian? galoisisabelian($G, \{flag\}$)
abelian number fields/ \mathbf{Q} galoissubcyclo($N, H, \{flag\}, \{v\}$)
query the galpol package galoisgetpol($a, b, \{s\}$)

Relative Number Fields (rnf)

Extension L/K is defined by $T \in K[x]$.
absolute equation of L rnfequation($nf, T, \{flag\}$)
is L/K abelian? rnfisabelian(nf, T)
relative nfalgtobasis rnfalgtobasis(rnf, x)
relative nfbasistoalg rnfbasistoalg(rnf, x)
relative idealhnf rnfidealhnf(rnf, x)
relative idealmul rnfidealmul(rnf, x, y)
relative idealtwoelt rnfidealtwoelt(rnf, x)

Lifts and Push-downs

absolute \rightarrow relative repres. for x rnfeltabstorel(rnf, x)
relative \rightarrow absolute repres. for x rnfeltreltoabs(rnf, x)
lift x to the relative field rnfeltup(rnf, x)
push x down to the base field rnfeltdown(rnf, x)
idem for x ideal: (rnfideal)reltoabs, abstorel, up, down

Norms

absolute norm of ideal x rnfidealnrmabs(rnf, x)
relative norm of ideal x rnfidealnrmrel(rnf, x)
solutions of $N_{K/\mathbf{Q}}(y) = x \in \mathbf{Z}$ bnfnisintnorm(bnf, x)
is $x \in \mathbf{Q}$ a norm from K ? bnfnisnorm($bnf, x, \{flag\}$)
initialize T for norm eq. solver rnfisnorminit($K, pol, \{flag\}$)
is $a \in K$ a norm from L ? rnfisnorm($T, a, \{flag\}$)

Maximal order \mathbf{Z}_L as a \mathbf{Z}_K -module

relative polred rnfpolred(nf, T)
relative polredabs rnfpolredabs(nf, T)
characteristic poly. of a mod T rnfcharpoly($nf, T, a, \{v\}$)
relative Dedekind criterion, prime pr rnfdedekind(nf, T, pr)
discriminant of relative extension rnfdisc(nf, T)
pseudo-basis of \mathbf{Z}_L rnfpsseudobasis(nf, T)
General \mathbf{Z}_K -modules: $M = [\text{matrix, vec. of ideals}] \subset L$
relative HNF / SNF nfhnf(nf, M), nfsnf
reduced basis for M rnflllgram(nf, T, M)
determinant of pseudo-matrix M rnfdet(nf, M)
Steinitz class of M rnfsteinitz(nf, M)
 \mathbf{Z}_K -basis of M if \mathbf{Z}_K -free, or 0 rnfhnfbasis(bnf, M)
 n -basis of M , or $(n + 1)$ -generating set rnfbasis(bnf, M)
is M a free \mathbf{Z}_K -module? rnfisfree(bnf, M)

Graphic Functions

crude graph of $expr$ between a and b plot($X = a, b, expr$)
High-resolution plot (immediate plot)
plot $expr$ between a and b ploth($X = a, b, expr, \{flag\}, \{n\}$)
plot points given by lists lx, ly plothraw($lx, ly, \{flag\}$)
terminal dimensions plotsizes()

Rectwindow functions

init window w , with size x, y plotinit(w, x, y)
erase window w plotkill(w)
copy w to w_2 with offset (dx, dy) plotcopy(w, w_2, dx, dy)
clips contents of w plotclip(w)
scale coordinates in w plotscale(w, x_1, x_2, y_1, y_2)
ploth in w plotrecth($w, X = a, b, expr, \{flag\}, \{n\}$)
plothraw in w plotrecthraw($w, data, \{flag\}$)
draw window w_1 at $(x_1, y_1), \dots$ plotdraw([$w_1, x_1, y_1], \dots$)

Low-level Rectwindow Functions

set current drawing color in w to c plotcolor(w, c)
current position of cursor in w plotcursor(w)
write s at cursor's position plotstring(w, s)
move cursor to (x, y) plotmove(w, x, y)
move cursor to $(x + dx, y + dy)$ plotrmove(w, dx, dy)
draw a box to (x_2, y_2) plotbox(w, x_2, y_2)
draw a box to $(x + dx, y + dy)$ plotrbox(w, dx, dy)
draw polygon plotlines($w, lx, ly, \{flag\}$)
draw points plotpoints(w, lx, ly)
draw line to $(x + dx, y + dy)$ plotrline(w, dx, dy)
draw point $(x + dx, y + dy)$ plotrpoint(w, dx, dy)
draw point $(x + dx, y + dy)$ plotrpoint(w, dx, dy)

Postscript Functions

as ploth psploth($X = a, b, expr, \{flag\}, \{n\}$)
as plothraw psplothraw($lx, ly, \{flag\}$)
as plotdraw psdraw([$w_1, x_1, y_1], \dots$)