

# PARI-GP Reference Card

(PARI-GP version 2.6.1)

Note: optional arguments are surrounded by braces {}.

To start the calculator, type its name in the terminal: **gp**

To exit **gp**, type **quit**, **\q**, or **<C-D>** at prompt.

## Help

describe function	? <i>function</i>
extended description	?? <i>keyword</i>
list of relevant help topics	??? <i>pattern</i>

## Input/Output

previous result, the result before	%, %', %'', etc.
<i>n</i> -th result since startup	% <i>n</i>
separate multiple statements on line	;
extend statement on additional lines	\
extend statements on several lines	{ <i>seq</i> <sub>1</sub> ; <i>seq</i> <sub>2</sub> ;
comment	/* ... */
one-line comment, rest of line ignored	\ \ ...

## Metacommands & Defaults

set default <i>d</i> to <i>val</i>	default({ <i>d</i> }, { <i>val</i> }, { <i>flag</i> })
toggle timer on/off	#
print time for last result	##
print defaults	\d
set debug level to <i>n</i>	\g <i>n</i>
set memory debug level to <i>n</i>	\gm <i>n</i>
set output mode (raw=0, default=1)	\o <i>n</i>
set <i>n</i> significant digits	\p <i>n</i>
set <i>n</i> terms in series	\ps <i>n</i>
quit GP	\q
print the list of PARI types	\t
print the list of user-defined functions	\u
read file into GP	\r <i>filename</i>

## Debugger / break loop

get out of break loop	break or <C-D>
go up <i>n</i> frames	dbg_up({ <i>n</i> })
examine object <i>o</i>	dbg_x( <i>o</i> )

## PARI Types & Input Formats

t_INT/t_REAL. Integers, Reals	$\pm n$ , $\pm n.ddd$
t_INTMOD. Integers modulo <i>m</i>	Mod( <i>n</i> , <i>m</i> )
t_FRAC. Rational Numbers	<i>n</i> / <i>m</i>
t_FFELT. Elt in finite field $F_q$	ffgen( <i>q</i> )
t_COMPLEX. Complex Numbers	$x + y * I$
t_PADIC. <i>p</i> -adic Numbers	$x + O(p^k)$
t_QUAD. Quadratic Numbers	$x + y * \text{quadgen}(D)$
t_POLMOD. Polynomials modulo <i>g</i>	Mod( <i>f</i> , <i>g</i> )
t_POL. Polynomials	$a * x^n + \dots + b$
t_SER. Power Series	$f + O(x^k)$
t_QFI/t_QFR. Imag/Real bin. quad. forms	Qfb( <i>a</i> , <i>b</i> , <i>c</i> , { <i>d</i> })
t_RFRAC. Rational Functions	<i>f</i> / <i>g</i>
t_VEC/t_COL. Row/Column Vectors	[ <i>x</i> , <i>y</i> , <i>z</i> ], [ <i>x</i> , <i>y</i> , <i>z</i> ]~
t_MAT. Matrices	[ <i>x</i> , <i>y</i> ; <i>z</i> , <i>t</i> ; <i>u</i> , <i>v</i> ]
t_LIST. Lists	List([ <i>x</i> , <i>y</i> , <i>z</i> ])
t_STR. Strings	"abc"

## Reserved Variable Names

$\pi = 3.14\dots$ , $\gamma = 0.57\dots$ , $C = 0.91\dots$	Pi, Euler, Catalan
square root of $-1$	I
big-oh notation	O

## Information about an Object

PARI type of object <i>x</i>	type( <i>x</i> )
length of <i>x</i> / size of <i>x</i> in memory	# <i>x</i> , sizebyte( <i>x</i> )
real or <i>p</i> -adic precision of <i>x</i>	precision( <i>x</i> ), padicprec

## Operators

basic operations	+, -, *, /, ^
i=i+1, i=i-1, i=i*j, ...	i++, i--, i*=j, ...
euclidean quotient, remainder	$x \backslash y$ , $x \setminus y$ , $x \% y$ , divrem( <i>x</i> , <i>y</i> )
shift <i>x</i> left or right <i>n</i> bits	$x < < n$ , $x > > n$ or shift( <i>x</i> , $\pm n$ )
comparison operators	<=, <, >=, >, ==, !=, ===, lex, cmp
boolean operators (or, and, not)	, &&, !
bit operations	bitand, bitneg, bitor, bitxor
sign of $x = -1, 0, 1$	sign( <i>x</i> )
maximum/minimum of <i>x</i> and <i>y</i>	max, min( <i>x</i> , <i>y</i> )
integer or real factorial of <i>x</i>	<i>x</i> ! or factorial( <i>x</i> )
derivative of <i>f</i> w.r.t. <i>x</i>	<i>f</i> '
apply differential operator	diffop
restore <i>x</i> as a formal variable	<i>x</i> =' <i>x</i>
simultaneous assignment $x \leftarrow v_1, y \leftarrow v_2$	[ <i>x</i> , <i>y</i> ] = v

## Select Components

<i>n</i> -th component of <i>x</i>	component( <i>x</i> , <i>n</i> )
<i>n</i> -th component of vector/list <i>x</i>	<i>x</i> [ <i>n</i> ]
components <i>a</i> , $a + 1, \dots, b$ of vector <i>x</i>	<i>x</i> [ <i>a</i> .. <i>b</i> ]
( <i>m</i> , <i>n</i> )-th component of matrix <i>x</i>	<i>x</i> [ <i>m</i> , <i>n</i> ]
row <i>m</i> or column <i>n</i> of matrix <i>x</i>	<i>x</i> [ <i>m</i> ,], <i>x</i> [, <i>n</i> ]
numerator/denominator of <i>x</i>	numerator( <i>x</i> ), denominator

## Conversions

to vector, matrix, set, list, string	Col/Vec, Mat, Set, List, Str
create PARI object ( <i>x</i> mod <i>y</i> )	Mod( <i>x</i> , <i>y</i> )
make <i>x</i> a polynomial of <i>v</i>	Pol( <i>x</i> , { <i>v</i> })
as Pol/Vec, starting with constant term	Polrev, Vecrev
make <i>x</i> a power series of <i>v</i>	Ser( <i>x</i> , { <i>v</i> })
string from bytes / from format+args	Strchr, Strprintf
convert <i>x</i> to simplest possible type	simplify( <i>x</i> )
object <i>x</i> with precision <i>n</i>	precision( <i>x</i> , <i>n</i> )

## Conjugates and Lifts

conjugate of a number <i>x</i>	conj( <i>x</i> )
conjugate vector of algebraic number <i>x</i>	conjvec( <i>x</i> )
norm of <i>x</i> , product with conjugate	norm( <i>x</i> )
square of $L^2$ norm of vector <i>x</i>	norml2( <i>x</i> )
lift of <i>x</i> from Mods	lift, centerlift( <i>x</i> )

## Lists, Sets & Sorting

sort <i>x</i> by <i>k</i> -th component	vecsort( <i>x</i> , { <i>k</i> }, { <i>fl</i> = 0})
min. <i>m</i> of <i>x</i> ( $m = x[i]$ ), max.	vecmin( <i>x</i> , {& <i>i</i> }), vecmax
does <i>y</i> belong to <i>x</i> , sorted wrt. <i>f</i>	vecsearch( <i>x</i> , <i>y</i> , { <i>f</i> })
<b>Sets</b> (= row vector of strings with strictly increasing entries)	
intersection of sets <i>x</i> and <i>y</i>	setintersect( <i>x</i> , <i>y</i> )
set of elements in <i>x</i> not belonging to <i>y</i>	setminus( <i>x</i> , <i>y</i> )
union of sets <i>x</i> and <i>y</i>	setunion( <i>x</i> , <i>y</i> )
does <i>y</i> belong to the set <i>x</i>	setsearch( <i>x</i> , <i>y</i> , { <i>flag</i> })
is <i>x</i> a set ?	setisset( <i>x</i> )
<b>Lists.</b> create empty list: $L = \text{List}()$	
append <i>x</i> to list <i>L</i>	listput( <i>L</i> , <i>x</i> , { <i>i</i> })
remove <i>i</i> -th component from list <i>L</i>	listpop( <i>L</i> , { <i>i</i> })
insert <i>x</i> in list <i>L</i> at position <i>i</i>	listinsert( <i>L</i> , <i>x</i> , <i>i</i> )
sort the list <i>L</i> in place	listsort( <i>L</i> , { <i>flag</i> })

## Programming

### Functions and closures

fun(vars) = my(local vars); *seq*

fun = (vars) -> my(local vars); *seq*

**Control Statements** (*X*: formal parameter in expression *seq*)

eval. <i>seq</i> for $a \leq X \leq b$	for( <i>X</i> = <i>a</i> , <i>b</i> , <i>seq</i> )
eval. <i>seq</i> for <i>X</i> dividing <i>n</i>	fordiv( <i>n</i> , <i>X</i> , <i>seq</i> )
eval. <i>seq</i> for primes $a \leq X \leq b$	forprime( <i>X</i> = <i>a</i> , <i>b</i> , <i>seq</i> )
eval. <i>seq</i> for $a \leq X \leq b$ stepping <i>s</i>	forstep( <i>X</i> = <i>a</i> , <i>b</i> , <i>s</i> , <i>seq</i> )
multivariable for	forvec( <i>X</i> = <i>v</i> , <i>seq</i> )
loop over partitions of <i>n</i>	forpart( <i>p</i> = <i>n</i> <i>seq</i> )
loop over vectors $v, q(v) \leq B, q > 0$	forqfvec( <i>v</i> , <i>q</i> , <i>b</i> , <i>seq</i> )
loop over subgrps <i>H</i> of abelian grp <i>G</i>	forsubgroup( <i>H</i> = <i>G</i> )
evaluate <i>seq</i> until $a \neq 0$	until( <i>a</i> , <i>seq</i> )
while $a \neq 0$ , evaluate <i>seq</i>	while( <i>a</i> , <i>seq</i> )
exit <i>n</i> innermost enclosing loops	break({ <i>n</i> })
start new iteration of <i>n</i> -th enclosing loop	next({ <i>n</i> })
return <i>x</i> from current subroutine	return({ <i>x</i> })
raise an exception	error()
if $a \neq 0$ , evaluate <i>seq</i> <sub>1</sub> , else <i>seq</i> <sub>2</sub>	if( <i>a</i> , { <i>seq</i> <sub>1</sub> }, { <i>seq</i> <sub>2</sub> })
try <i>seq</i> <sub>1</sub> , evaluate <i>seq</i> <sub>2</sub> on error	iferr( <i>seq</i> <sub>1</sub> , <i>E</i> , <i>seq</i> <sub>2</sub> )
select from <i>v</i> according to <i>f</i>	select( <i>f</i> , <i>v</i> )
apply <i>f</i> to all entries in <i>v</i>	apply( <i>f</i> , <i>v</i> )

### Input/Output

print with/without \n, T <sub>E</sub> X format	print, print1, printtex
formatted printing	printf()
write <i>args</i> to file	write, writel, writetex( <i>file</i> , <i>args</i> )
write <i>x</i> in binary format	writebin( <i>file</i> , <i>x</i> )
read file into GP	read({ <i>file</i> })
read file, return as vector of lines	readvec({ <i>file</i> })
read a string from keyboard	input()

### Interface with User and System

allocates a new stack of <i>s</i> bytes	allocatemem({ <i>s</i> })
alias <i>old</i> to <i>new</i>	alias( <i>new</i> , <i>old</i> )
install function from library	install( <i>f</i> , <i>code</i> , { <i>gpf</i> }, { <i>lib</i> })
execute system command <i>a</i>	system( <i>a</i> )
as above, feed result to GP	extern( <i>a</i> )
as above, return GP string	externstr( <i>a</i> )
get \$VAR from environment	getenv("VAR")
measure time in ms.	gettime()
timeout command after <i>s</i> seconds	alarm( <i>s</i> , <i>expr</i> )

## Iterations, Sums & Products

numerical integration	intnum( <i>X</i> = <i>a</i> , <i>b</i> , <i>expr</i> , { <i>flag</i> })
sum <i>expr</i> over divisors of <i>n</i>	sumdiv( <i>n</i> , <i>X</i> , <i>expr</i> )
sumdiv, with <i>expr</i> multiplicative	sumdivmult( <i>n</i> , <i>X</i> , <i>expr</i> )
sum $X = a$ to $X = b$ , initialized at <i>x</i>	sum( <i>X</i> = <i>a</i> , <i>b</i> , <i>expr</i> , { <i>x</i> })
sum of series <i>expr</i>	suminf( <i>X</i> = <i>a</i> , <i>expr</i> )
sum of alternating/positive series	sumalt, sumpos
sum of series using intnum	sumnum
product $a \leq X \leq b$ , initialized at <i>x</i>	prod( <i>X</i> = <i>a</i> , <i>b</i> , <i>expr</i> , { <i>x</i> })
product over primes $a \leq X \leq b$	prodeuler( <i>X</i> = <i>a</i> , <i>b</i> , <i>expr</i> )
infinite product $a \leq X \leq \infty$	prodinf( <i>X</i> = <i>a</i> , <i>expr</i> )
real root of <i>expr</i> between <i>a</i> and <i>b</i>	solve( <i>X</i> = <i>a</i> , <i>b</i> , <i>expr</i> )

## Random Numbers

random integer/prime in $[0, N[$	random( <i>N</i> ), randomprime
get/set random seed	getrand, setrand( <i>s</i> )

Vectors & Matrices

dimensions of matrix $x$	<code>matsize(<math>x</math>)</code>
concatenation of $x$ and $y$	<code>concat(<math>x, \{y\}</math>)</code>
extract components of $x$	<code>vecextract(<math>x, y, \{z\}</math>)</code>
transpose of vector or matrix $x$	<code>mattranspose(<math>x</math>)</code> or <code>x-</code>
adjoint of the matrix $x$	<code>matadjoint(<math>x</math>)</code>
eigenvectors/values of matrix $x$	<code>mateigen(<math>x</math>)</code>
characteristic/minimal polynomial of $x$	<code>charpoly(<math>x</math>)</code> , <code>minpoly</code>
trace/determinant of matrix $x$	<code>trace(<math>x</math>)</code> , <code>matdet</code>
Frobenius form of $x$	<code>matfrobenius(<math>x</math>)</code>
QR decomposition	<code>matqr(<math>x</math>)</code>

Constructors & Special Matrices

row vec. of $expr$ eval'd at $1 \leq i \leq n$	<code>vector(<math>n, \{i\}, \{expr\}</math>)</code>
col. vec. of $expr$ eval'd at $1 \leq i \leq n$	<code>vectorv(<math>n, \{i\}, \{expr\}</math>)</code>
matrix $1 \leq i \leq m, 1 \leq j \leq n$	<code>matrix(<math>m, n, \{i\}, \{j\}, \{expr\}</math>)</code>
define matrix by blocks	<code>matconcat(<math>B</math>)</code>
diagonal matrix with diagonal $x$	<code>matdiagonal(<math>x</math>)</code>
$n \times n$ identity matrix	<code>matid(<math>n</math>)</code>
Hessenberg form of square matrix $x$	<code>mathess(<math>x</math>)</code>
$n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$	<code>mathilbert(<math>n</math>)</code>
companion matrix to polynomial $x$	<code>matcompanion(<math>x</math>)</code>
Sylvester matrix of $x$	<code>polsylvestermatrix(<math>x</math>)</code>

Gaussian elimination

kernel of matrix $x$	<code>matker(<math>x, \{flag\}</math>)</code>
intersection of column spaces of $x$ and $y$	<code>matintersect(<math>x, y</math>)</code>
solve $M * X = B$ ( $M$ invertible)	<code>matsolve(<math>M, B</math>)</code>
as solve, modulo $D$ (col. vector)	<code>matolvemod(<math>M, D, B</math>)</code>
one sol of $M * X = B$	<code>matinverseimage(<math>M, B</math>)</code>
basis for image of matrix $x$	<code>matimage(<math>x</math>)</code>
supplement columns of $x$ to get basis	<code>mat supplement(<math>x</math>)</code>
rows, cols to extract invertible matrix	<code>matindexrank(<math>x</math>)</code>
rank of the matrix $x$	<code>matrank(<math>x</math>)</code>

Lattices & Quadratic Forms

upper triangular Hermite Normal Form	<code>mathnf(<math>x</math>)</code>
HNF of $x$ where $d$ is a multiple of $\det(x)$	<code>mathnfmod(<math>x, d</math>)</code>
elementary divisors of $x$	<code>matsnf(<math>x</math>)</code>
LLL-algorithm applied to columns of $x$	<code>qflll(<math>x, \{flag\}</math>)</code>
like <code>qflll</code> , $x$ is Gram matrix of lattice	<code>qflllgram(<math>x, \{flag\}</math>)</code>
LLL-reduced basis for kernel of $x$	<code>matkerint(<math>x</math>)</code>
$\mathbf{Z}$ -lattice $\longleftrightarrow$ $\mathbf{Q}$ -vector space	<code>matrixqz(<math>x, p</math>)</code>
signature of quad form ${}^t y * x * y$	<code>qf sign(<math>x</math>)</code>
decomp into squares of ${}^t y * x * y$	<code>qfgaussred(<math>x</math>)</code>
eigenvals/eigenvecs for real symmetric $x$	<code>qfjacobi(<math>x</math>)</code>
find up to $m$ sols of ${}^t y * x * y \leq b$	<code>qfminim(<math>x, b, m</math>)</code>
perfection rank of $x$	<code>qfperfection(<math>x</math>)</code>
$v, v[i] :=$ number of sols of ${}^t y * x * y = i$	<code>qfrep(<math>x, B, \{flag\}</math>)</code>
automorphism group of $q$	<code>qfauto(<math>q</math>)</code>
find isomorphism between $q$ and $Q$	<code>qfisom(<math>q, Q</math>)</code>

Formal & p-adic Series

truncate power series or $p$ -adic number	<code>truncate(<math>x</math>)</code>
valuation of $x$ at $p$	<code>valuation(<math>x, p</math>)</code>

Dirichlet and Power Series

Taylor expansion around 0 of $f$ w.r.t. $x$	<code>taylor(<math>f, x</math>)</code>
$\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$	<code>serconvol(<math>a, b</math>)</code>
$f = \sum a_k t^k$ from $\sum (a_k / k!) t^k$	<code>serlaplace(<math>f</math>)</code>
reverse power series $F$ so $F(f(x)) = x$	<code>serreverse(<math>f</math>)</code>
Dirichlet series multiplication / division	<code>dirmul, dirdiv(<math>x, y</math>)</code>
Dirichlet Euler product ( $b$ terms)	<code>direuler(<math>p = a, b, expr</math>)</code>

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Polynomials & Rational Functions

degree of $f$	<code>poldegree(<math>f</math>)</code>
coeff. of degree $n$ of $f$ , leading coeff.	<code>polcoeff(<math>f, n</math>)</code> , <code>pollead</code>
gcd of coefficients of $f$	<code>content(<math>f</math>)</code>
replace $x$ by $y$	<code>subst(<math>f, x, y</math>)</code>
evaluate $f$ replacing vars by their value	<code>eval(<math>f</math>)</code>
replace polynomial expr. $T(x)$ by $y$ in $f$	<code>substpol(<math>f, T, y</math>)</code>
replace $x_1, \dots, x_n$ by $y_1, \dots, y_n$ in $f$	<code>substvec(<math>f, x, y</math>)</code>
discriminant of polynomial $f$	<code>poldisc(<math>f</math>)</code>
resultant $R = \text{Res}_v(f, g)$	<code>polresultant(<math>f, g, \{v\}</math>)</code>
$[u, v, R], xu + yv = \text{Res}_v(f, g)$	<code>polresultanttext(<math>x, y, \{v\}</math>)</code>
derivative of $f$ w.r.t. $x$	<code>deriv(<math>f, \{x\}</math>)</code>
formal integral of $f$ w.r.t. $x$	<code>intformal(<math>f, \{x\}</math>)</code>
formal sum of $f$ w.r.t. $x$	<code>sumformal(<math>f, \{x\}</math>)</code>
reciprocal poly $x^{\deg f} f(1/x)$	<code>polrecip(<math>f</math>)</code>
interpol. pol. eval. at $a$	<code>polinterpolate(<math>X, \{Y\}, \{a\}, \{&amp;e\}</math>)</code>
initialize $t$ for Thue equation solver	<code>thueinit(<math>f</math>)</code>
solve Thue equation $f(x, y) = a$	<code>thue(<math>t, a, \{sol\}</math>)</code>

Roots and Factorization

number of real roots of $f, a < x \leq b$	<code>polsturm(<math>f, \{a\}, \{b\}</math>)</code>
complex roots of $f$	<code>polroots(<math>f</math>)</code>
symmetric powers of roots of $f$ up to $n$	<code>polsym(<math>f, n</math>)</code>
factor $f$	<code>factor(<math>f, \{lim\}</math>)</code>
factor $f$ mod $p$ / roots	<code>factormod(<math>f, p</math>)</code> , <code>polrootsmod</code>
factor $f$ over $\mathbf{F}_{p^a}$ / roots	<code>factorff(<math>f, p, a</math>)</code> , <code>polrootsff</code>
factor $f$ over $\mathbf{Q}_p$ / roots	<code>factorpadic(<math>f, p, r</math>)</code> , <code>polrootspadic</code>
find irreducible $T \in \mathbf{F}_p[x], \deg T = n$	<code>ffinit(<math>p, n, \{x\}</math>)</code>
$\#\{\text{monic irred. } T \in \mathbf{F}_q[x], \deg T = n\}$	<code>ffnbirred(<math>q, n</math>)</code>
$p$ -adic root of $f$ cong. to $a$ mod $p$	<code>padicappr(<math>f, a</math>)</code>
Newton polygon of $f$ for prime $p$	<code>newtonpoly(<math>f, p</math>)</code>
extensions of $\mathbf{Q}_p$ of degree $N$	<code>padicfields(<math>p, N</math>)</code>

Special Polynomials

$n$ -th cyclotomic polynomial in var. $v$	<code>polcyclo(<math>n, \{v\}</math>)</code>
$d$ -th degree subfield of $\mathbf{Q}(\zeta_n)$	<code>polsubcyclo(<math>n, d, \{v\}</math>)</code>
$P_n, T_n/U_n, H_n$	<code>pollegendre, polchebyshev, polhermite</code>

Transcendental and  $p$ -adic Functions

real, imaginary part of $x$	<code>real(<math>x</math>)</code> , <code>imag(<math>x</math>)</code>
absolute value, argument of $x$	<code>abs(<math>x</math>)</code> , <code>arg(<math>x</math>)</code>
square/ $n$ th root of $x$	<code>sqrtn(<math>x</math>)</code> , <code>sqrtn(<math>x, n, \{&amp;z\}</math>)</code>
trig functions	<code>sin, cos, tan, cotan</code>
inverse trig functions	<code>asin, acos, atan</code>
hyperbolic functions	<code>sinh, cosh, tanh</code>
inverse hyperbolic functions	<code>asinh, acosh, atanh</code>
exponential / natural log of $x$	<code>exp, log</code>
Euler $\Gamma$ function, $\log \Gamma, \Gamma'/\Gamma$	<code>gamma, lngamma, psi</code>
incomplete gamma function ( $y = \Gamma(s)$ )	<code>incgam(<math>s, x, \{y\}</math>)</code>
exponential integral $\int_x^\infty e^{-t}/t dt$	<code>eint1(<math>x</math>)</code>
error function $2/\sqrt{\pi} \int_x^\infty e^{-t^2} dt$	<code>erfc(<math>x</math>)</code>
dilogarithm of $x$	<code>dilog(<math>x</math>)</code>
$m$ -th polylogarithm of $x$	<code>polylog(<math>m, x, \{flag\}</math>)</code>
$U$ -confluent hypergeometric function	<code>hyperu(<math>a, b, u</math>)</code>
Bessel $J_n(x), J_{n+1/2}(x)$	<code>besselj(<math>n, x</math>)</code> , <code>besseljh(<math>n, x</math>)</code>
Bessel $I_\nu, K_\nu, H_\nu^1, H_\nu^2, N_\nu$	<code>(bessel)i, k, h1, h2, n</code>
Lambert $W: x$ s.t. $xe^x = y$	<code>lambertw(<math>y</math>)</code>
Teichmuller character of $p$ -adic $x$	<code>teichmuller(<math>x</math>)</code>

Elementary Arithmetic Functions

vector of binary digits of $ x $	<code>binary(<math>x</math>)</code>
bit number $n$ of integer $x$	<code>bittest(<math>x, n</math>)</code>
Hamming weight of integer $x$	<code>hammingweight(<math>x</math>)</code>
ceiling/floor/fractional part	<code>ceil, floor, frac</code>
round $x$ to nearest integer	<code>round(<math>x, \{&amp;e\}</math>)</code>
truncate $x$	<code>truncate(<math>x, \{&amp;e\}</math>)</code>
gcd/LCM of $x$ and $y$	<code>gcd(<math>x, y</math>)</code> , <code>lcm(<math>x, y</math>)</code>
gcd of entries of a vector/matrix	<code>content(<math>x</math>)</code>

Primes and Factorization

add primes in $v$ to prime table	<code>addprimes(<math>v</math>)</code>
Chebyshev $\pi(x)$ , $n$ -th prime $p_n$	<code>primepi(<math>x</math>)</code> , <code>prime(<math>n</math>)</code>
vector of first $n$ primes	<code>primes(<math>n</math>)</code>
smallest prime $\geq x$	<code>nextprime(<math>x</math>)</code>
largest prime $\leq x$	<code>precprime(<math>x</math>)</code>
factorization of $x$	<code>factor(<math>x, \{lim\}</math>)</code>
$n = df^2$ , $d$ squarefree/fundamental	<code>core(<math>n, \{fl\}</math>)</code> , <code>coredisc</code>
recover $x$ from its factorization	<code>factorback(<math>f, \{e\}</math>)</code>

Divisors

number of prime divisors $\omega(n)$ / $\Omega(n)$	<code>omega(<math>n</math>)</code> , <code>bigomega</code>
divisors of $n$ / number of divisors $\tau(n)$	<code>divisors(<math>n</math>)</code> , <code>numdiv</code>
sum of ( $k$ -th powers of) divisors of $n$	<code>sigma(<math>n, \{k\}</math>)</code>

Special Functions and Numbers

binomial coefficient $\binom{x}{y}$	<code>binomial(<math>x, y</math>)</code>
Bernoulli number $B_n$ as real/rational	<code>bernreal(<math>n</math>)</code> , <code>bernfrac</code>
Bernoulli polynomial $B_n(x)$	<code>bernpol(<math>n, \{x\}</math>)</code>
$n$ -th Fibonacci number	<code>fibonacci(<math>n</math>)</code>
Stirling numbers $s(n, k)$ and $S(n, k)$	<code>stirling(<math>n, k, \{flag\}</math>)</code>
number of partitions of $n$	<code>numbpart(<math>n</math>)</code>
Möbius $\mu$ -function	<code>moebius(<math>x</math>)</code>
Hilbert symbol of $x$ and $y$ (at $p$ )	<code>hilbert(<math>x, y, \{p\}</math>)</code>
Kronecker-Legendre symbol $(\frac{x}{y})$	<code>kronecker(<math>x, y</math>)</code>
Dedekind sum $s(h, k)$	<code>sumdedekind(<math>h, k</math>)</code>

Multiplicative groups  $(\mathbf{Z}/N\mathbf{Z})^*$ ,  $\mathbf{F}_q^*$

Euler $\phi$ -function	<code>eulerphi(<math>x</math>)</code>
multiplicative order of $x$ (divides $o$ )	<code>znorder(<math>x, \{o\}</math>)</code> , <code>fforder</code>
primitive root mod $q$ / $x$ .mod	<code>znprimroot(<math>q</math>)</code> , <code>ffprimroot(<math>x</math>)</code>
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	<code>znstar(<math>n</math>)</code>
discrete logarithm of $x$ in base $g$	<code>znlog(<math>x, g, \{o\}</math>)</code> , <code>fflog</code>

Miscellaneous

integer square / $n$ -th root of $x$	<code>sqrtnint(<math>x</math>)</code> , <code>sqrtnint(<math>x, n</math>)</code>
solve $z \equiv x$ and $z \equiv y$	<code>chinese(<math>x, y</math>)</code>
minimal $u, v$ so $xu + yv = \text{gcd}(x, y)$	<code>gcdext(<math>x, y</math>)</code>
continued fraction of $x$	<code>contfrac(<math>x, \{b\}, \{lmax\}</math>)</code>
last convergent of continued fraction $x$	<code>contfracpnqn(<math>x</math>)</code>
rational approximation to $x$	<code>bestappr(<math>x, k</math>)</code> , <code>bestapprPade</code>

True-False Tests

is $x$ the disc. of a quadratic field?	<code>isfundamental(<math>x</math>)</code>
is $x$ a prime?	<code>isprime(<math>x</math>)</code>
is $x$ a strong pseudo-prime?	<code>ispseudoprime(<math>x</math>)</code>
is $x$ square-free?	<code>issquarefree(<math>x</math>)</code>
is $x$ a square?	<code>issquare(<math>x, \{&amp;n\}</math>)</code>
is $x$ a perfect power?	<code>ispower(<math>x, \{k\}, \{&amp;n\}</math>)</code>
is $pol$ irreducible?	<code>polisirreducible(<math>pol</math>)</code>

Based on an earlier version by Joseph H. Silverman  
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# PARI-GP Reference Card (2)

(PARI-GP version 2.6.1)

## Elliptic Curves

Elliptic curve initially given by 5-tuple  $v = [a_1, a_2, a_3, a_4, a_6]$ . Initialize *ell* struct  $E = \text{ellinit}(v, \{Domain\})$ . Points are  $[x, y]$ , the origin is  $[0]$ . Struct members accessed as  $E.member$ :

- All domains: **E.a1,a2,a3,a4,a6, b2,b4,b6,b8, c4,c6, disc, j**
  - $E$  defined over **R** or **C**
    - $x$ -coords. of points of order 2 **E.roots**
    - periods / quasi-periods **E.omega,E.eta**
    - volume of complex lattice **E.area**
  - $E$  defined over  **$\mathbf{Q}_p$** 
    - residual characteristic **E.p**
    - If  $|j|_p > 1$ : Tate's  $[u^2, u, q, [a, b]]$  **E.tate**
  - $E$  defined over  **$\mathbf{F}_q$** 
    - characteristic **E.p**
    - $\#E(\mathbf{F}_q)$ /cyclic structure/generators **E.no, E.cyc, E.gen**
  - $E$  defined over  **$\mathbf{Q}$** 
    - generators of  $E(\mathbf{Q})$  (require **elldata**) **E.gen**
    - $[a_1, a_2, a_3, a_4, a_6]$  from  $j$ -invariant **ellfromj(j)**
    - change curve  $E$  using  $v = [u, r, s, t]$  **ellchangecurve(E, v)**
    - change point  $z$  using  $v = [u, r, s, t]$  **ellchangepoint(z, v)**
    - add points  $P + Q / P - Q$  **elladd(E, P, Q), ellsub**
    - negate point **ellneg(E, P)**
    - compute  $n \cdot z$  **ellmul(E, z, n)**
    - $n$ -division polynomial  $f_n(x)$  **elldivpol(E, n, {x})**
    - check if  $z$  is on  $E$  **ellisoncurve(E, z)**
    - order of torsion point  $z$  **ellorder(E, z)**
    - $y$ -coordinates of point(s) for  $x$  **ellordinate(E, x)**
    - point  $[\wp(z), \wp'(z)]$  corresp. to  $z$  **ellztopoint(E, z)**
    - complex  $z$  such that  $p = [\wp(z), \wp'(z)]$  **ellpointtoz(E, p)**
- Curves over finite fields, Pairings**
- random point on  $E$  **random(E)**
  - $\#E(\mathbf{F}_q)$  **ellcard(E)**
  - structure  $\mathbf{Z}/d_1\mathbf{Z} \times \mathbf{Z}/d_2\mathbf{Z}$  of  $E(\mathbf{F}_q)$  **ellgroup(E)**
  - Weil pairing of  $m$ -torsion pts  $x, y$  **ellweilpairing(E, x, y, m)**
  - Tate pairing of  $x, y$ ;  $x$   $m$ -torsion **elltatepairing(E, x, y, m)**
  - Discrete log, find  $n$  s.t.  $P = [n]Q$  **ellllog(E, P, Q, {ord})**
- Curves over  $\mathbf{Q}$  and the  $L$ -function**
- canonical bilinear form taken at  $z_1, z_2$  **ellbil(E, z\_1, z\_2)**
  - canonical height of  $z$  **ellheight(E, z, {flag})**
  - height regulator matrix for pts in  $x$  **ellheightmatrix(E, x)**
  - cond, min mod, Tamagawa num  $[N, v, c]$  **ellglobalred(E)**
  - reduction of  $y^2 + Qy = P$  (genus 2) **genus2red(Q, P, {p})**
  - Kodaira type of  $p$ -fiber of  $E$  **elllocalred(E, p)**
  - minimal model of  $E/\mathbf{Q}$  **ellminimalmodel(E, {&v})**
  - $p$ -th coeff  $a_p$  of  $L$ -function,  $p$  prime **ellap(E, p)**
  - $k$ -th coeff  $a_k$  of  $L$ -function **ellak(E, k)**
  - vector of first  $n$   $a_k$ 's in  $L$ -function **elllan(E, n)**
  - $L(E, s)$  **elllseries(E, s)**
  - $L^{(r)}(E, 1)$  **ellL1(E, r)**
  - return a Heegner point on  $E$  of rank 1 **ellheegner(E)**
  - order of vanishing at 1 **ellanalyticrank(E, {eps})**
  - root number for  $L(E, \cdot)$  at  $p$  **ellrootno(E, {p})**
  - torsion subgroup with generators **elltors(E)**
  - modular parametrization of  $E$  **elltaniyama(E)**

## Elldata package, Cremona's database:

db code  $\leftrightarrow$  *[conductor, class, index]* **ellconvertname(s)**  
generators of Mordell-Weil group **ellgenerators(E)**  
look up  $E$  in database **ellidentify(E)**  
all curves matching criterion **ellsearch(N)**  
loop over curves with cond. from  $a$  to  $b$  **forell(E, a, b, seq)**

## Elliptic & Modular Functions

$w = [\omega_1, \omega_2]$  or *ell* struct **(E.omega)**,  $\tau = \omega_1/\omega_2$ .  
arithmetic-geometric mean **agm(x, y)**  
elliptic  $j$ -function  $1/q + 744 + \dots$  **ellj(x)**  
Weierstrass  $\sigma/\wp/\zeta$  function **ellsigma(w, z), ellwp, ellzeta**  
periods/quasi-periods **ellperiods(E, {flag}), elleta(w)**  
 $(2i\pi/\omega_2)^k E_k(\tau)$  **elleisnum(w, k, {flag})**  
modified Dedekind  $\eta$  func.  $\prod(1 - q^n)$  **eta(x, {flag})**  
Jacobi sine theta function **theta(q, z)**  
 $k$ -th derivative at  $z=0$  of **theta(q, z)** **thetanullk(q, k)**  
Weber's  $f$  functions **weber(x, {flag})**  
Riemann's zeta  $\zeta(s) = \sum n^{-s}$  **zeta(s)**

## Binary Quadratic Forms

create  $ax^2 + bxy + cy^2$  (distance  $d$ ) **Qfb(a, b, c, {d})**  
reduce  $x$  ( $s = \sqrt{D}$ ,  $t = [s]$ ) **qfbred(x, {flag}, {D}, {l}, {s})**  
composition of forms  $x*y$  or **qfbnucomp(x, y, l)**  
 $n$ -th power of form  $x^n$  or **qfbnupow(x, n)**  
composition without reduction **qfbcomprow(x, y)**  
 $n$ -th power without reduction **qfbpowrow(x, n)**  
prime form of disc.  $x$  above prime  $p$  **qfbprimeform(x, p)**  
class number of disc.  $x$  **qfbclassno(x)**  
Hurwitz class number of disc.  $x$  **qfbhclassno(x)**  
Solve  $Q(x, y) = p$  in integers,  $p$  prime **qfbsolve(Q, p)**

## Quadratic Fields

quadratic number  $\omega = \sqrt{x}$  or  $(1 + \sqrt{x})/2$  **quadgen(x)**  
minimal polynomial of  $\omega$  **quadpoly(x)**  
discriminant of  **$\mathbf{Q}(\sqrt{D})$**  **quaddisc(x)**  
regulator of real quadratic field **quadregulator(x)**  
fundamental unit in real  **$\mathbf{Q}(x)$**  **quadunit(x)**  
class group of  **$\mathbf{Q}(\sqrt{D})$**  **quadclassunit(D, {flag}, {t})**  
Hilbert class field of  **$\mathbf{Q}(\sqrt{D})$**  **quadhilbert(D, {flag})**  
ray class field modulo  $f$  of  **$\mathbf{Q}(\sqrt{D})$**  **quadray(D, f, {flag})**

## General Number Fields: Initializations

A number field  $K$  is given by a monic irreducible  $f \in \mathbf{Z}[X]$ .

init number field structure *nf* **nfinit(f, {flag})**

### nf members:

polynomial defining *nf*,  $f(\theta) = 0$  **nf.pol**  
number of real/complex places **nf.r1/r2/sign**  
discriminant of *nf* **nf.disc**  
 $T_2$  matrix **nf.t2**  
vector of roots of  $f$  **nf.roots**  
integral basis of  $\mathbf{Z}_K$  as powers of  $\theta$  **nf.zk**  
different **nf.diff**  
codifferent **nf.codiff**  
index **nf.index**  
recompute *nf* using current precision **nfnewprec(nf)**  
init relative *rnf* given by  $g = 0$  over  $K$  **rnfinit(nf, g)**  
init *bnf* structure **bnfinit(f, {flag})**

### bnf members: same as *nf*, plus

underlying *nf* **bnf.nf**  
classgroup **bnf.clgp**  
regulator **bnf.reg**  
fundamental units **bnf.fu**  
torsion units **bnf.tu**  
compute a *bnf* from small *bnf* **bnfinit(sbnf)**  
add  $S$ -class group and units, yield *bnf* **bnfsunit(nf, S)**  
init class field structure *bnr* **bnrinit(bnf, m, {flag})**  
**bnr members:** same as *bnf*, plus  
underlying *bnf* **bnr.bnf**  
big ideal structure **bnr.bid**  
modulus **bnr.mod**  
structure of  **$(\mathbf{Z}_K/m)^*$**  **bnr.zkst**

## Basic Number Field Arithmetic (nf)

Elements are **t\_INT, t\_FRAC, t\_POL, t\_POLMOD**, or **t\_COL** (on integral basis *nf.zk*). Basic operations (prefix **nfelt**): (**nfelt**)**add, mul, pow, div, diveuc, mod, divrem, val, trace, norm**  
express  $x$  on integer basis **nfalgtobasis(nf, x)**  
express element  $x$  as a polmod **nfbasistoalg(nf, x)**  
reverse polmod  $a = A(X) \bmod T(X)$  **modreverse(a)**  
integral basis of field def. by  $f = 0$  **nfbasis(f)**  
field discriminant of field  $f = 0$  **nfdisc(f)**  
smallest poly defining  $f = 0$  (slow) **polredabs(f, {flag})**  
small poly defining  $f = 0$  (fast) **polredbest(f, {flag})**  
are fields  $f = 0$  and  $g = 0$  isomorphic? **nfisism(f, g)**  
is field  $f = 0$  a subfield of  $g = 0$ ? **nfisincl(f, g)**  
compositum of  $f = 0, g = 0$  **polcompositum(f, g, {flag})**  
subfields (of degree  $d$ ) of *nf* **nfsubfields(nf, {d})**  
roots of unity in *nf* **nfrootsof1(nf)**  
roots of  $g$  belonging to *nf* **nfroots({nf}, g)**  
factor  $g$  in *nf* **nfactor(nf, g)**  
factor  $g$  mod prime *pr* in *nf* **nfactormod(nf, g, pr)**  
conjugates of a root  $\theta$  of *nf* **nfgaloisconj(nf, {flag})**  
apply Galois automorphism  $s$  to  $x$  **nfgaloisapply(nf, s, x)**  
quadratic Hilbert symbol (at  $p$ ) **nfhilbert(nf, a, b, {p})**

### Linear and algebraic relations

poly of degree  $\leq k$  with root  $x \in \mathbf{C}$  **algdep(x, k)**  
alg. dep. with pol. coeffs for series  $s$  **seralgdep(s, x, y)**  
small linear rel. on coords of vector  $x$  **lindep(x)**  
**Dedekind Zeta Function  $\zeta_K$ , Hecke  $L$  series**  
 $\zeta_K$  as Dirichlet series,  $N(I) < b$  **dirzetak(nf, b)**  
init *nfz* for field  $f = 0$  **zetakinit(f)**  
compute  $\zeta_K(s)$  **zetak(nfz, s, {flag})**  
Artin root number of  $K$  **bnrrootnumber(bnr, chi, {flag})**  
 $L(1, \chi)$ , for all  $\chi$  trivial on  $H$  **bnrL1(bnr, {H}, {flag})**

## Class Groups & Units (bnf, bnr)

$a_1, \{a_2\}, \{a_3\}$  usually *bnr, subgp* or *bnf, module, {subgp}*  
remove GRH assumption from *bnf* **bnfcertify(bnf)**  
expo. of ideal  $x$  on class gp **bnfisprincipal(bnf, x, {flag})**  
expo. of ideal  $x$  on ray class gp **bnrisprincipal(bnr, x, {flag})**  
expo. of  $x$  on fund. units **bnfisunit(bnf, x)**  
as above for  $S$ -units **bnfissunit(bnf, x)**  
signs of real embeddings of *bnf.fu* **bnfsignunit(bnf)**  
narrow class group **bnfnarrow(bnf)**

Class Field Theory

ray class number for mod.  $m$                     `bnrclassno(bnf, m)`  
discriminant of class field ext                `bnrdisc(a1, {a2}, {a3})`  
ray class numbers,  $l$  list of mods            `bnrclassnolist(bnf, l)`  
discriminants of class fields `bnrdisclist(bnf, l, {arch}, {flag})`  
decode output from `bnrdisclist`    `bnfdecodemodule(nf, fa)`  
is modulus the conductor?    `bnrisconductor(a1, {a2}, {a3})`  
conductor of character  $chi$     `bnrconductorofchar(bnr, chi)`  
conductor of extension    `bnrconductor(a1, {a2}, {a3}, {flag})`  
conductor of extension def. by  $g$     `rnfconductor(bnf, g)`  
Artin group of ext. def'd by  $g$     `rnfnormgroup(bnr, g)`  
subgroups of  $bnr$ , index  $\leq b$     `subgrouplist(bnr, b, {flag})`  
rel. eq. for class field def'd by  $sub$     `rnfkummer(bnr, sub, {d})`  
same, using Stark units (real field)    `bnrstark(bnr, sub, {flag})`

**Ideals:** elements, primes, or matrix of generators in HNF  
is  $id$  an ideal in  $nf$  ?                    `nfisideal(nf, id)`  
is  $x$  principal in  $bnf$  ?                `bnfisprincipal(bnf, x)`  
give  $[a, b]$ , s.t.  $a\mathbf{Z}_K + b\mathbf{Z}_K = x$     `idealtwoelt(nf, x, {a})`  
put ideal  $a$  ( $a\mathbf{Z}_K + b\mathbf{Z}_K$ ) in HNF form    `idealhnf(nf, a, {b})`  
norm of ideal  $x$                     `idealnrm(nf, x)`  
minimum of ideal  $x$  (direction  $v$ )    `idealmin(nf, x, v)`  
LLL-reduce the ideal  $x$  (direction  $v$ )    `idealred(nf, x, {v})`

Ideal Operations

add ideals  $x$  and  $y$                     `idealadd(nf, x, y)`  
multiply ideals  $x$  and  $y$                 `idealmul(nf, x, y, {flag})`  
intersection of ideals  $x$  and  $y$     `idealintersect(nf, x, y, {flag})`  
 $n$ -th power of ideal  $x$                 `idealpow(nf, x, n, {flag})`  
inverse of ideal  $x$                     `idealinv(nf, x)`  
divide ideal  $x$  by  $y$                     `idealdiv(nf, x, y, {flag})`  
Find  $(a, b) \in x \times y, a + b = 1$     `idealaddtoone(nf, x, {y})`  
coprime integral  $A, B$  such that  $x = A/B$     `idealnumden(nf, x)`

Primes and Multiplicative Structure

factor ideal  $x$  in  $nf$                     `idealfactor(nf, x)`  
expand ideal factorization in  $nf$     `idealfactorback(nf, f, e)`  
decomposition of prime  $p$  in  $nf$             `idealprimedec(nf, p)`  
valuation of  $x$  at prime ideal  $pr$     `idealval(nf, x, pr)`  
weak approximation theorem in  $nf$     `idealchinese(nf, x, y)`  
give  $bid$  =structure of  $(\mathbf{Z}_K/id)^*$     `idealstar(nf, id, {flag})`  
discrete log of  $x$  in  $(\mathbf{Z}_K/bid)^*$     `ideallog(nf, x, bid)`  
`idealstar` of all ideals of norm  $\leq b$     `ideallist(nf, b, {flag})`  
add Archimedean places    `ideallistarch(nf, b, {ar}, {flag})`  
init `prmod` structure                    `nfmodprinit(nf, pr)`  
kernel of matrix  $M$  in  $(\mathbf{Z}_K/pr)^*$     `nfkernelmodpr(nf, M, prmod)`  
solve  $Mx = B$  in  $(\mathbf{Z}_K/pr)^*$     `nfsolvemodpr(nf, M, B, prmod)`

Galois theory over  $\mathbf{Q}$

Galois group of field  $\mathbf{Q}[x]/(f)$             `polgalois(f)`  
initializes a Galois group structure  $G$     `galoisinit(pol, {den})`  
action of  $p$  in nfgaloisconj form    `galoispermtopol(G, {p})`  
identify as abstract group                `galoisidentify(G)`  
export a group for GAP/MAGMA    `galoisexport(G, {flag})`  
subgroups of the Galois group  $G$             `galoissubgroups(G)`  
is subgroup  $H$  normal?                    `galoisisnormal(G, H)`  
subfields from subgroups    `galoissubfields(G, {flag}, {v})`  
fixed field                    `galoisfixedfield(G, perm, {flag}, {v})`  
Frobenius at maximal ideal  $P$             `idealfrobenius(nf, G, P)`  
ramification groups at  $P$                 `idealramgroups(nf, G, P)`

PARI-GP Reference Card (2)

(PARI-GP version 2.6.1)

is  $G$  abelian?                    `galoisisabelian(G, {flag})`  
abelian number fields/ $\mathbf{Q}$             `galoissubcyclo(N, H, {flag}, {v})`  
query the galpol package                `galoisgetpol(a, b, {s})`

Relative Number Fields (rnf)

Extension  $L/K$  is defined by  $T \in K[x]$ .  
absolute equation of  $L$                 `rnfequation(nf, T, {flag})`  
is  $L/K$  abelian?                    `rnfisabelian(nf, T)`  
relative nfgaltobasis                `rnfalgtobasis(rnf, x)`  
relative nfbasistoalg                `rnfbasistoalg(rnf, x)`  
relative idealhnf                    `rnfidealhnf(rnf, x)`  
relative idealmul                    `rnfidealmul(rnf, x, y)`  
relative idealtwoelt                `rnfidealtwoelt(rnf, x)`

Lifts and Push-downs

absolute  $\rightarrow$  relative repres. for  $x$     `rnfeltabstorel(rnf, x)`  
relative  $\rightarrow$  absolute repres. for  $x$     `rnfeltreltoabs(rnf, x)`  
lift  $x$  to the relative field                `rnfeltup(rnf, x)`  
push  $x$  down to the base field            `rnfeltdown(rnf, x)`  
idem for  $x$  ideal: (`rnfideal`)`reltoabs`, `abstorel`, `up`, `down`

Norms

absolute norm of ideal  $x$                 `rnfidealnrmabs(rnf, x)`  
relative norm of ideal  $x$                 `rnfidealnrmrel(rnf, x)`  
solutions of  $N_{K/\mathbf{Q}}(y) = x \in \mathbf{Z}$             `bnfisintnorm(bnf, x)`  
is  $x \in \mathbf{Q}$  a norm from  $K$ ?                `bnfisnorm(bnf, x, {flag})`  
initialize  $T$  for norm eq. solver    `rnfisnorminit(K, pol, {flag})`  
is  $a \in K$  a norm from  $L$ ?                `rnfisnorm(T, a, {flag})`

Maximal order  $\mathbf{Z}_L$  as a  $\mathbf{Z}_K$ -module

relative polred                    `rnfpolred(nf, T)`  
relative polredabs                    `rnfpolredabs(nf, T)`  
characteristic poly. of  $a$  mod  $T$     `rnfcharpoly(nf, T, a, {v})`  
relative Dedekind criterion, prime  $pr$     `rnfdedekind(nf, T, pr)`  
discriminant of relative extension    `rnfdisc(nf, T)`  
pseudo-basis of  $\mathbf{Z}_L$                     `rnfpsseudobasis(nf, T)`  
**General  $\mathbf{Z}_K$ -modules:**  $M = [\text{matrix, vec. of ideals}] \subset L$   
relative HNF / SNF                    `nfhnf(nf, M), nfsnf`  
reduced basis for  $M$                     `rnflllgram(nf, T, M)`  
determinant of pseudo-matrix  $M$     `rnfdet(nf, M)`  
Steinitz class of  $M$                     `rnfsteinitz(nf, M)`  
 $\mathbf{Z}_K$ -basis of  $M$  if  $\mathbf{Z}_K$ -free, or 0    `rnfhnfbasis(bnf, M)`  
 $n$ -basis of  $M$ , or  $(n + 1)$ -generating set    `rnfbasis(bnf, M)`  
is  $M$  a free  $\mathbf{Z}_K$ -module?                `rnfisfree(bnf, M)`

Graphic Functions

crude graph of  $expr$  between  $a$  and  $b$     `plot(X = a, b, expr)`  
**High-resolution plot** (immediate plot)  
plot  $expr$  between  $a$  and  $b$     `plotth(X = a, b, expr, {flag}, {n})`  
plot points given by lists  $lx, ly$     `plotthraw(lx, ly, {flag})`  
terminal dimensions                    `plotsizes()`

Rectwindow functions

init window  $w$ , with size  $x, y$             `plotinit(w, x, y)`  
erase window  $w$                     `plotkill(w)`  
copy  $w$  to  $w_2$  with offset  $(dx, dy)$     `plotcopy(w, w2, dx, dy)`  
clips contents of  $w$                     `plotclip(w)`  
scale coordinates in  $w$                 `plotscale(w, x1, x2, y1, y2)`  
`plotth` in  $w$                     `plotrecth(w, X = a, b, expr, {flag}, {n})`  
`plotthraw` in  $w$                     `plotrecthraw(w, data, {flag})`  
draw window  $w_1$  at  $(x_1, y_1), \dots$     `plotdraw([w1, x1, y1], \dots)`

Low-level Rectwindow Functions

set current drawing color in  $w$  to  $c$     `plotcolor(w, c)`  
current position of cursor in  $w$     `plotcursor(w)`  
write  $s$  at cursor's position            `plotstring(w, s)`  
move cursor to  $(x, y)$                     `plotmove(w, x, y)`  
move cursor to  $(x + dx, y + dy)$     `plotrmove(w, dx, dy)`  
draw a box to  $(x_2, y_2)$                 `plotbox(w, x2, y2)`  
draw a box to  $(x + dx, y + dy)$     `plotrbox(w, dx, dy)`  
draw polygon                    `plotlines(w, lx, ly, {flag})`  
draw points                    `plotpoints(w, lx, ly)`  
draw line to  $(x + dx, y + dy)$     `plotrline(w, dx, dy)`  
draw point  $(x + dx, y + dy)$     `plotrpoint(w, dx, dy)`  
draw point  $(x + dx, y + dy)$     `plotrpoint(w, dx, dy)`

Postscript Functions

as `plotth`                    `psplotth(X = a, b, expr, {flag}, {n})`  
as `plotthraw`                    `psplotthraw(lx, ly, {flag})`  
as `plotdraw`                    `psdraw([w1, x1, y1], \dots)`